(Orbital) Angular Momentum and GPDs in Lattice QCD

David Richards

Jefferson Laboratory

- ***Hadron Structure from Lattice QCD***
  - ***Anatomy of a calculation - Form factors***
- ***Generalized Parton Distributions***
  - **Quark Spin**
    Transversity -> TMDs - Berni Musch
  - ***Quark orbital angular momentum***
  - ***3D picture of nucleon***
- ***Next challenge - Flavor-singlet***
- ***Outlook***

GaryFest, Oct. 28-29, 2010

LHP Collaboration

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(LHPC)

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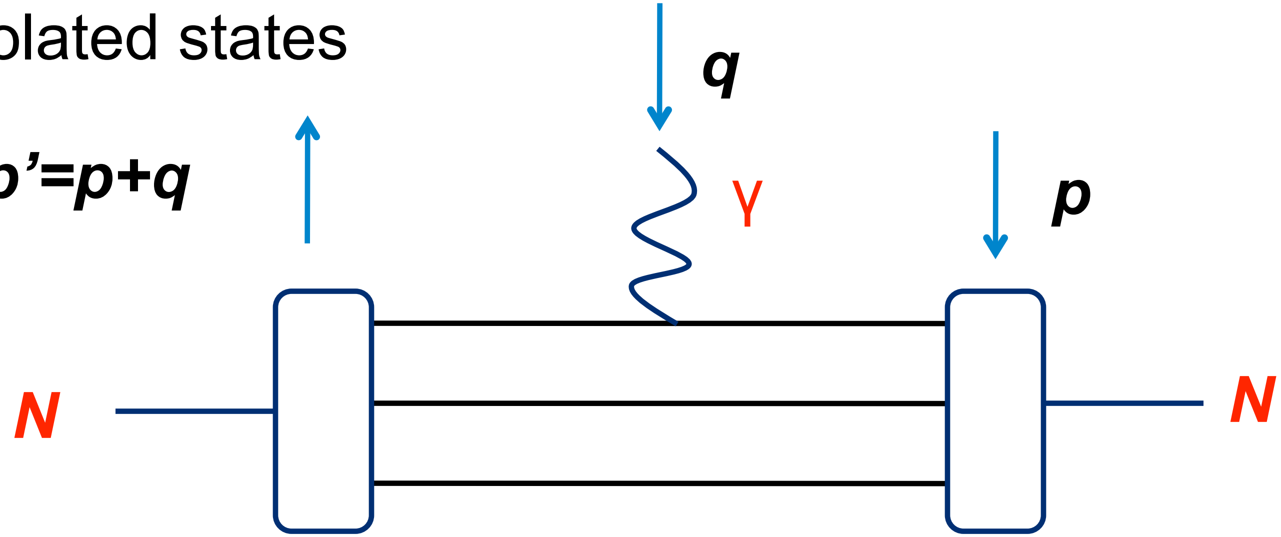
National Taiwan University, Taipei 10617, Taiwan, R.O.C.

(Dated: June 29, 2010)

Anatomy of a Calculation - I

- Lattice QCD computes the matrix elements between isolated states

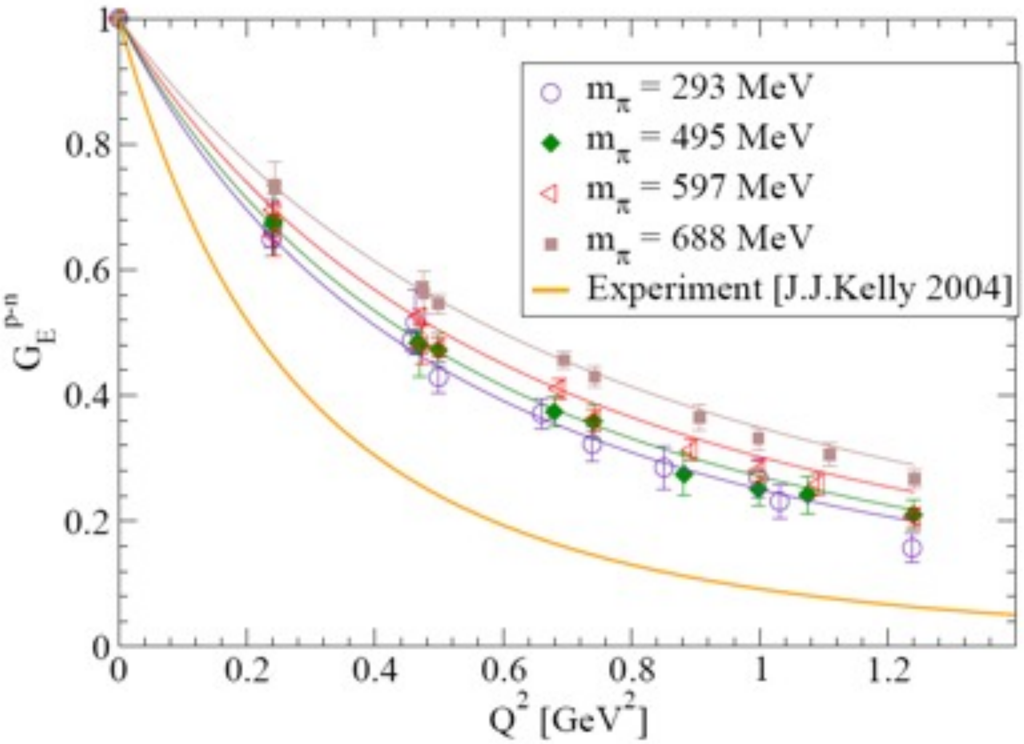


$$\Gamma_{3\mathrm{pt}}(\vec{p},\vec{q};t_f,t) = \sum_{\vec{x},\vec{y}} \langle N_1(\vec{x},t_f) V_\mu(\vec{y},t) N_2(0)\rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}}$$

Complete set of states

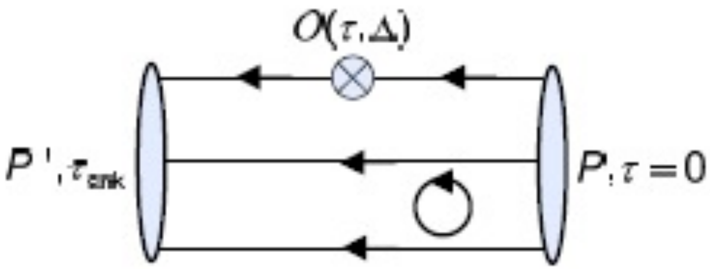
$$\langle N \mid V_\mu \mid N\rangle_\mu(q) = \bar{u}_N(p')\left[F_1(q^2) + \sigma_{\mu\nu}q_\nu \frac{F_2(q^2)}{2M}\right] u_N(p)$$

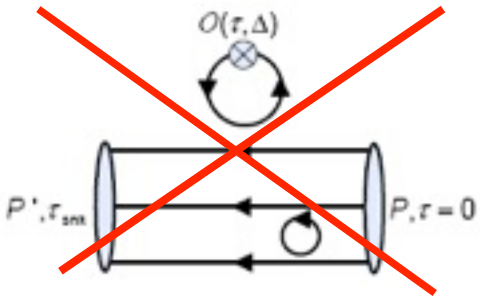
Isovector Form Factor



J.D.Bratt et al (LHPC), arXiv:0810.1933

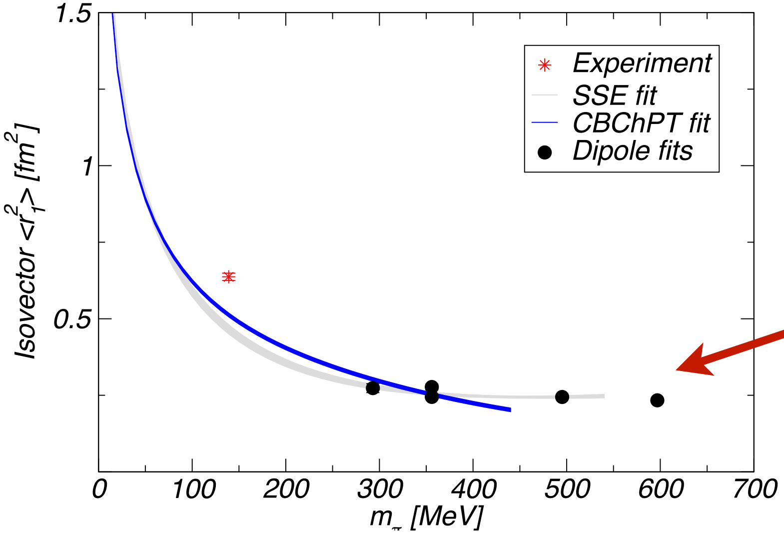
Euclidean lattice: *form factors in space-like region*





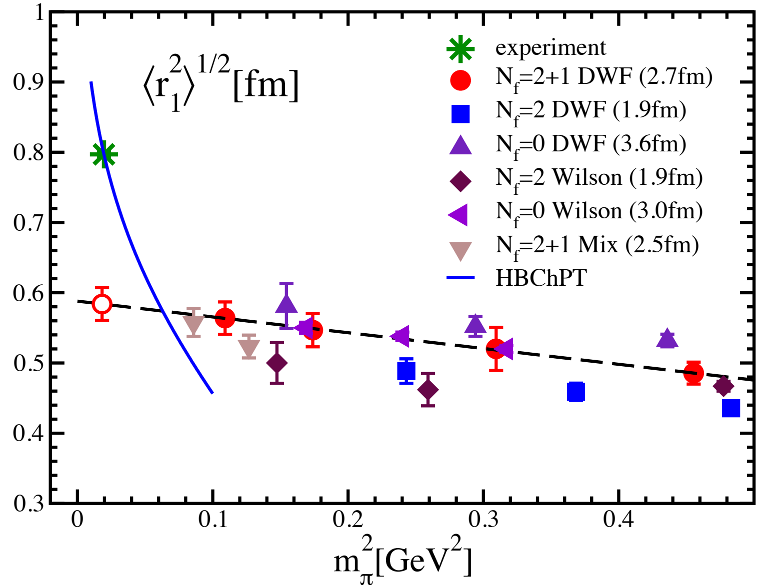
DWF valence/Asqtad sea

Nucleon Form Factors - III



LHPC, arXiv:1001.3620

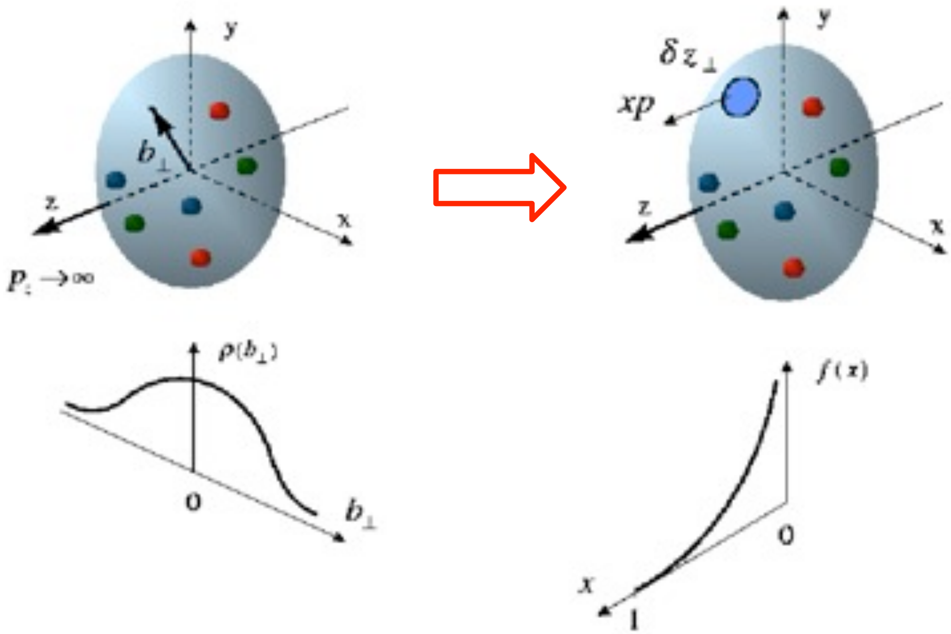
Dipole fits at each pion mass



RBC/UKQCD, arXiv:0904.2039

- Form factors well-described by dipole fits, or SSE at small Q2
- Non-analytic terms clear to reach physical value
- Notable finite-size effects in Dirac form factor

Different Regimes in Different Experiments



Form Factors
transverse quark distribution in Coordinate space

Structure Functions
longitudinal quark distribution in momentum space

Moments of Structure Functions

- **Describe distribution of longitudinal momentum and spin in proton**
- **Matrix elements of light-cone correlation functions**

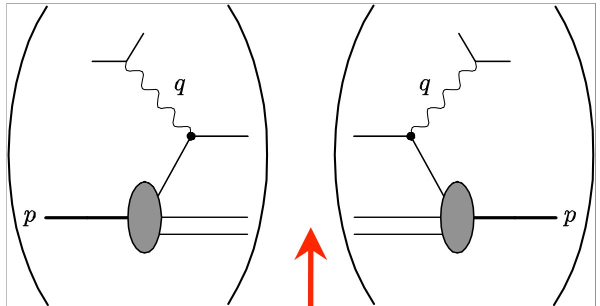
$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}\left(-\frac{\lambda}{2}n\right) nP e^{-ig\int_{\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} \psi\left(\frac{\lambda}{2}n\right)$$

- **Expand *O(x)* around light-cone**

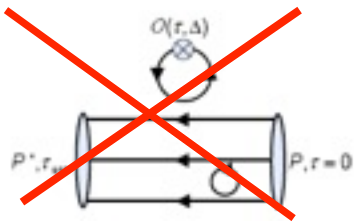
$$O_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \bar{\psi}_q \gamma_5 \gamma^{\{\mu_1} iD^{\mu_2} \ldots D^{\mu_n\}} \psi_q$$

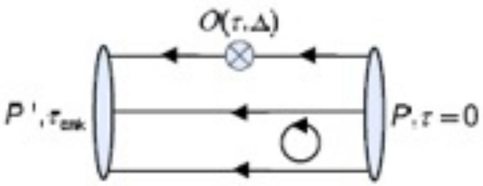
- **Diagonal matrix element**

$$\langle P|O_q^{\{\mu_1\ldots\mu_n\}}|P\rangle \simeq \int dx\, x^{n-1} q(x)$$



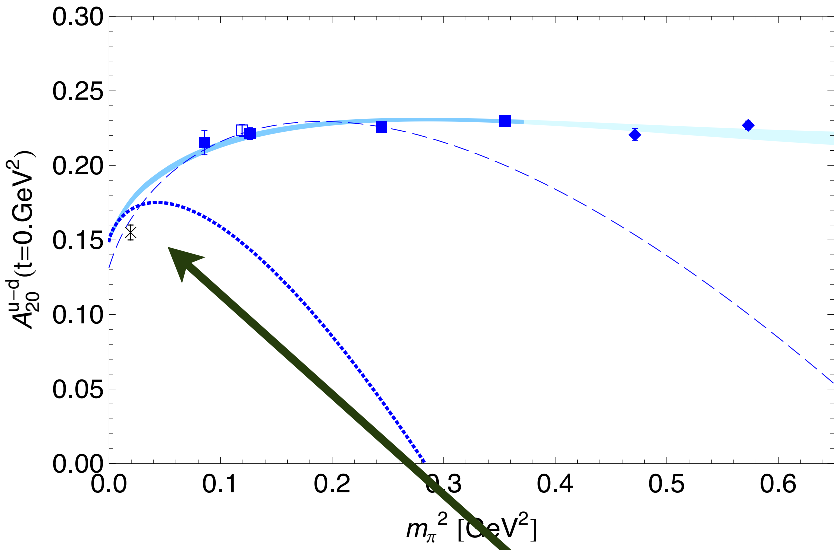
Dominated by lightest state





Iso-vector Momentum Fraction

Isovector momentum fraction $\langle x \rangle_{u-d}$



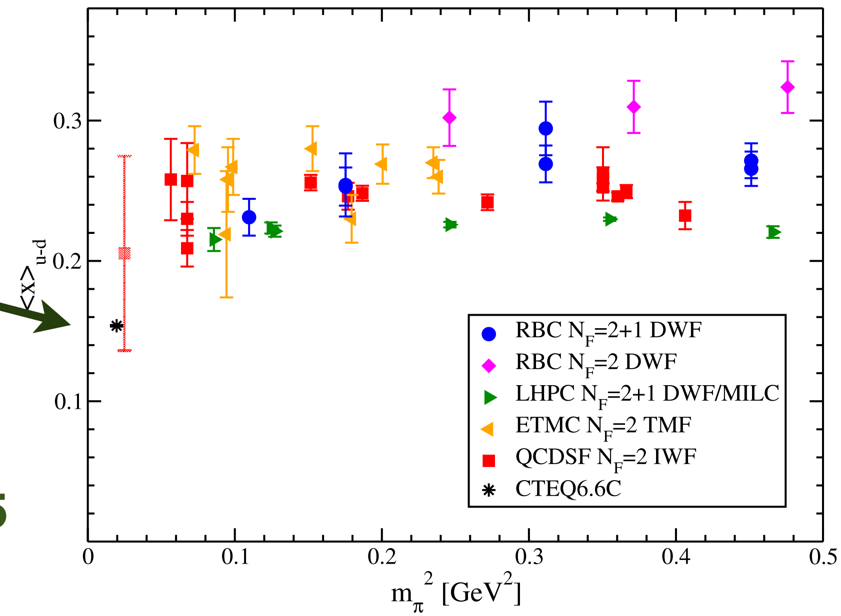
Covariant BChPT

HBChPT

LHPC,arXiv:1001.3620 (PRD, to appear)

HB limit of BChPT

CTEQ6



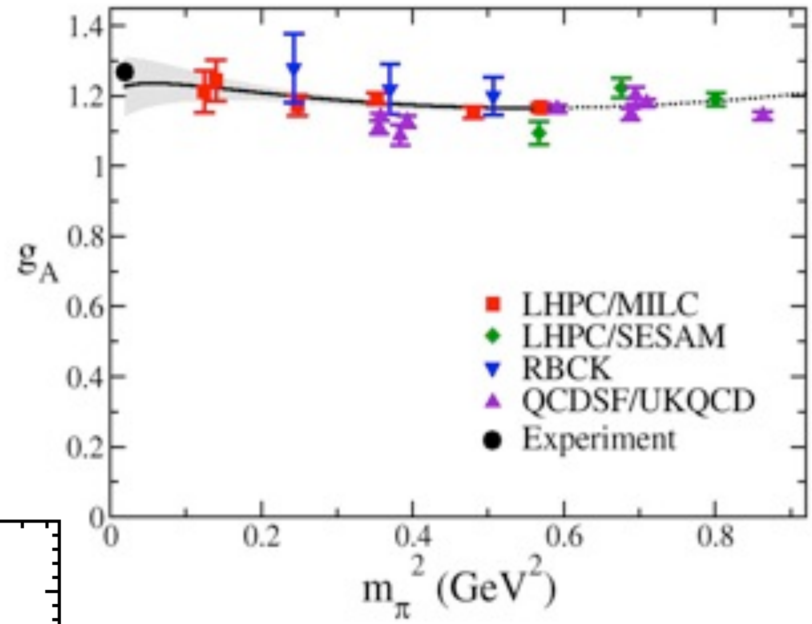
Dru Renner, arXiv:1002.0925

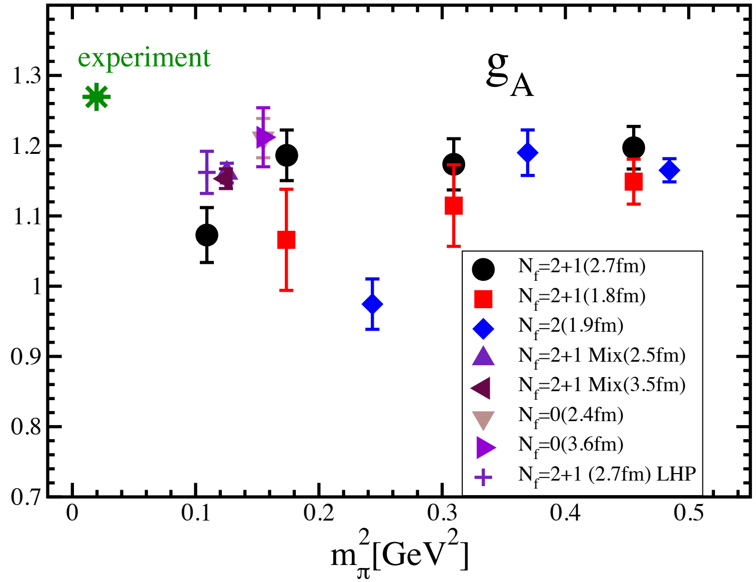
Nucleon Axial-Vector Charge - I

Nucleon's axial-vector charge g_A: benchmark of lattice QCD

- **Hybrid lattice QCD at m_π down to 350 MeV**
- **Finite-volume chiral-perturbation theory**

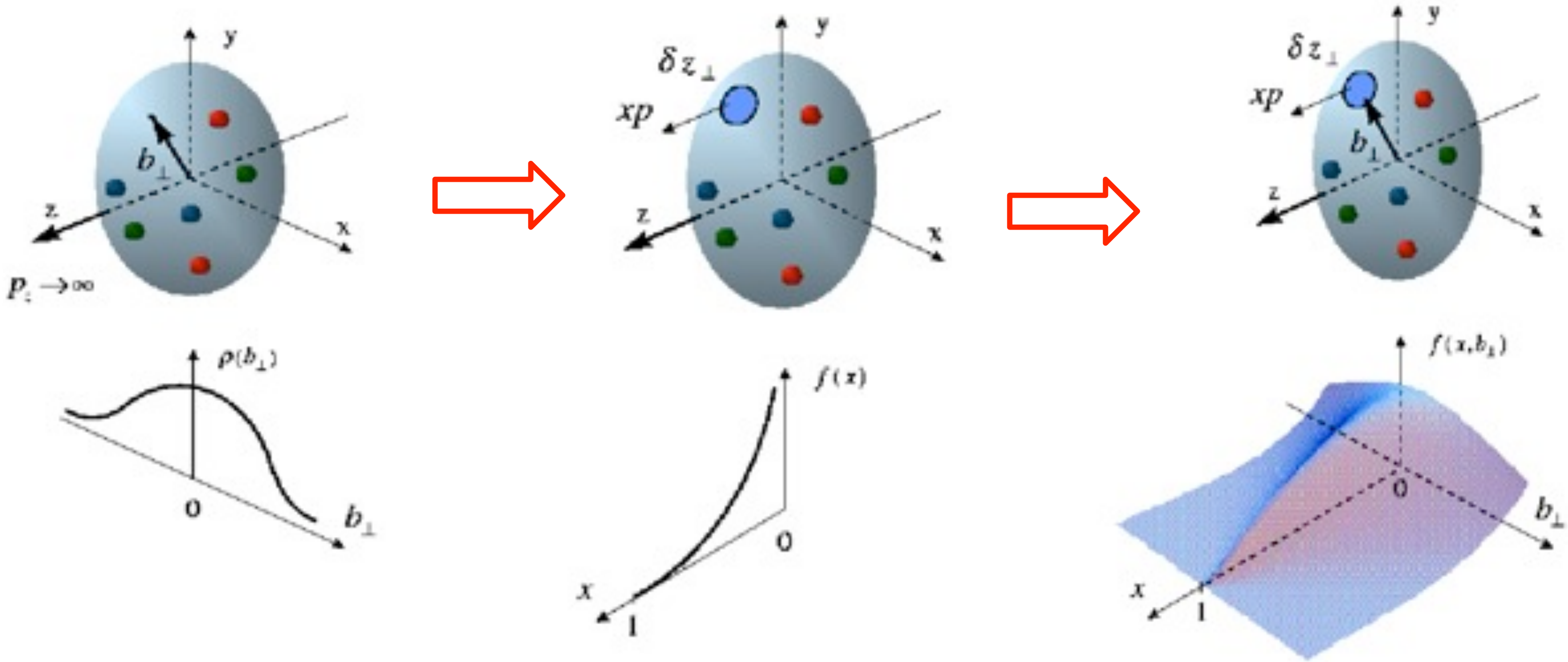
LHPC, PRL 96 (2006), 052001





RBC/UKQCD, 2+ 1 flavor DWF

Different Regimes in Different Experiments

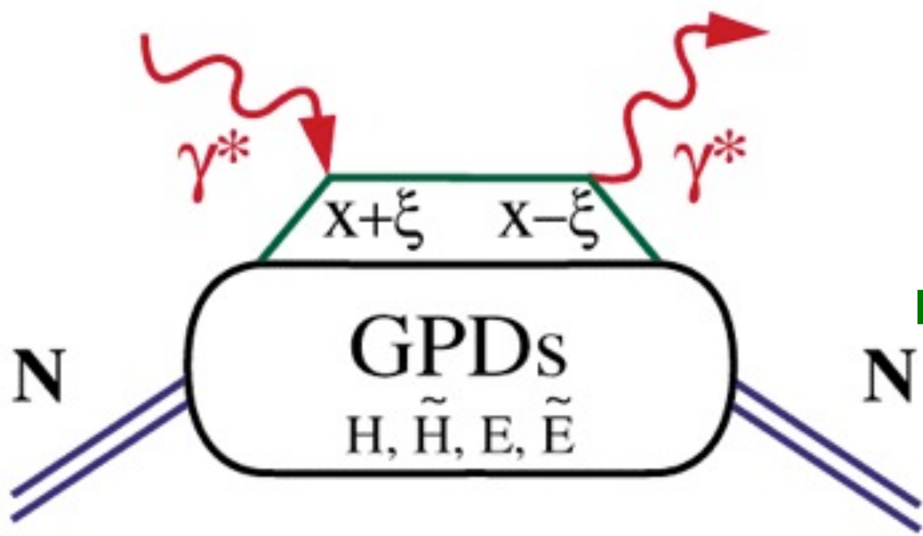


Form Factors
transverse quark distribution in Coordinate space

Structure Functions
longitudinal quark distribution in momentum space

GPDs
Fully-correlated quark distribution in both coordinate and momentum space

Generalized Parton Distributions (GPDs)



D. Muller *et al* (1994), X. Ji & A. Radyushkin (1996)

- Matrix elements of light-cone correlation functions

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}\left(-\frac{\lambda}{2}n\right) nP e^{-ig\int_{\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} \psi\left(\frac{\lambda}{2}n\right)$$

- Expand *O(x)* around light-cone

$$O_q^{\{\mu_1\mu_2\cdots\mu_n\}} = \bar{\psi}_q \gamma^{\{\mu_1} i D^{\mu_2} \ldots D^{\mu_n\}} \psi_q$$

LHPC, QCDSF, 2003

- *Off-forward* matrix element

Co-efficient of ξ^i

$$\langle P'|O_q^{\{\mu_1\cdots\mu_n\}}|P\rangle \simeq \int dx\, x^{n-1}[H(x,\xi,t), E(x,\xi,t)]$$

$$\longrightarrow A_{ni}(t), B_{ni}(t), C_n(t), \tilde{A}_{ni}(t), \tilde{B}_{ni}(t), \tilde{C}_n(t)$$

GPDs and Orbital Angular Momentum

- **Form factors of energy momentum tensor - *quark and gluon angular momentum***

"$\bar{q}\gamma_\mu D_\nu q$"

$$\frac{1}{2} = \sum_q J^q + J^g$$

X.D. Ji, PRL 78, 610 (1997)

$$= \frac{1}{2}\left\{\sum_q \left(A^q_{20}(t=0) + B^q_{20}(t=0)\right) + A^g_{20}(t=0) + B^g_{20}(t=0)\right\}$$

$$\sum_q \left(\frac{1}{2}\Delta\Sigma^q + L^q\right)$$

Decomposition

- **Gauge-invariant**
- **Renormalization-scale dependent**
- **Handle on Quark orbital angular momentum**

Mathur et al., *Phys.Rev. D62 (2000) 114504*

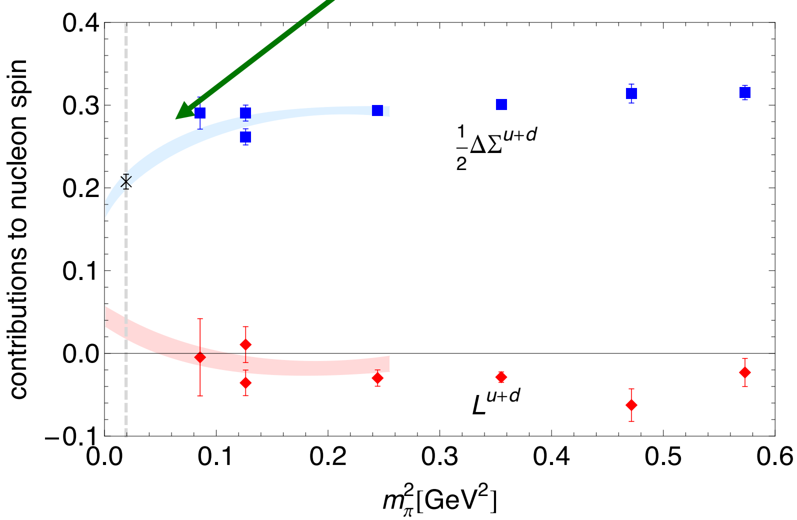
Origin of Nucleon Spin

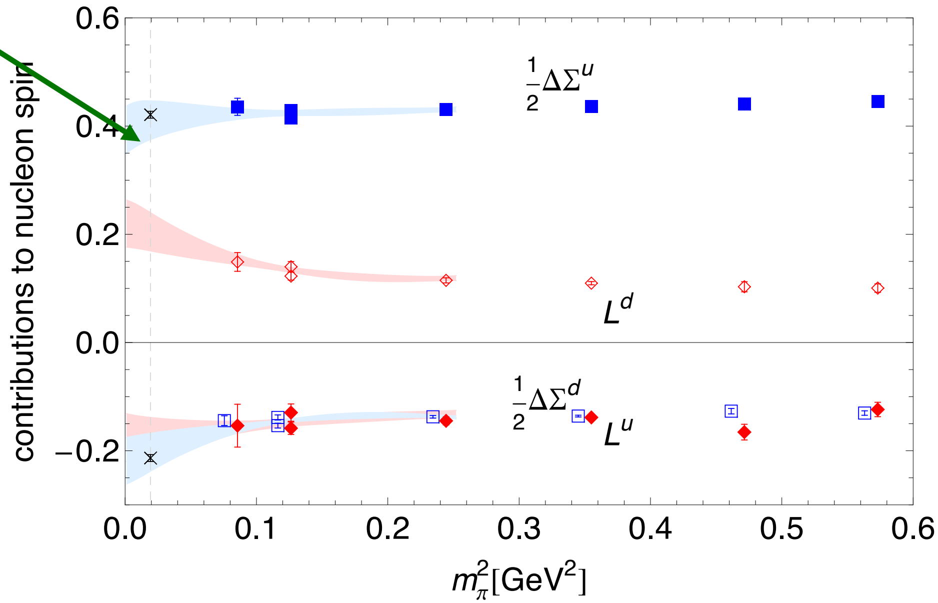
$$J^q = 1/2\left(A^q_{20}(t=0) + B^q_{20}(t-0)\right)$$
$$\Delta\Sigma^q/2 = \tilde{A}^q_{10}(t=0)/2$$
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^{u+d} + L^{u+d} + J^g$$

- **Total orbital angular momentum carried by quarks small**
- **Orbital angular momentum carried by individual quark flavours substantial.**

HERMES, PRD75 (2007)

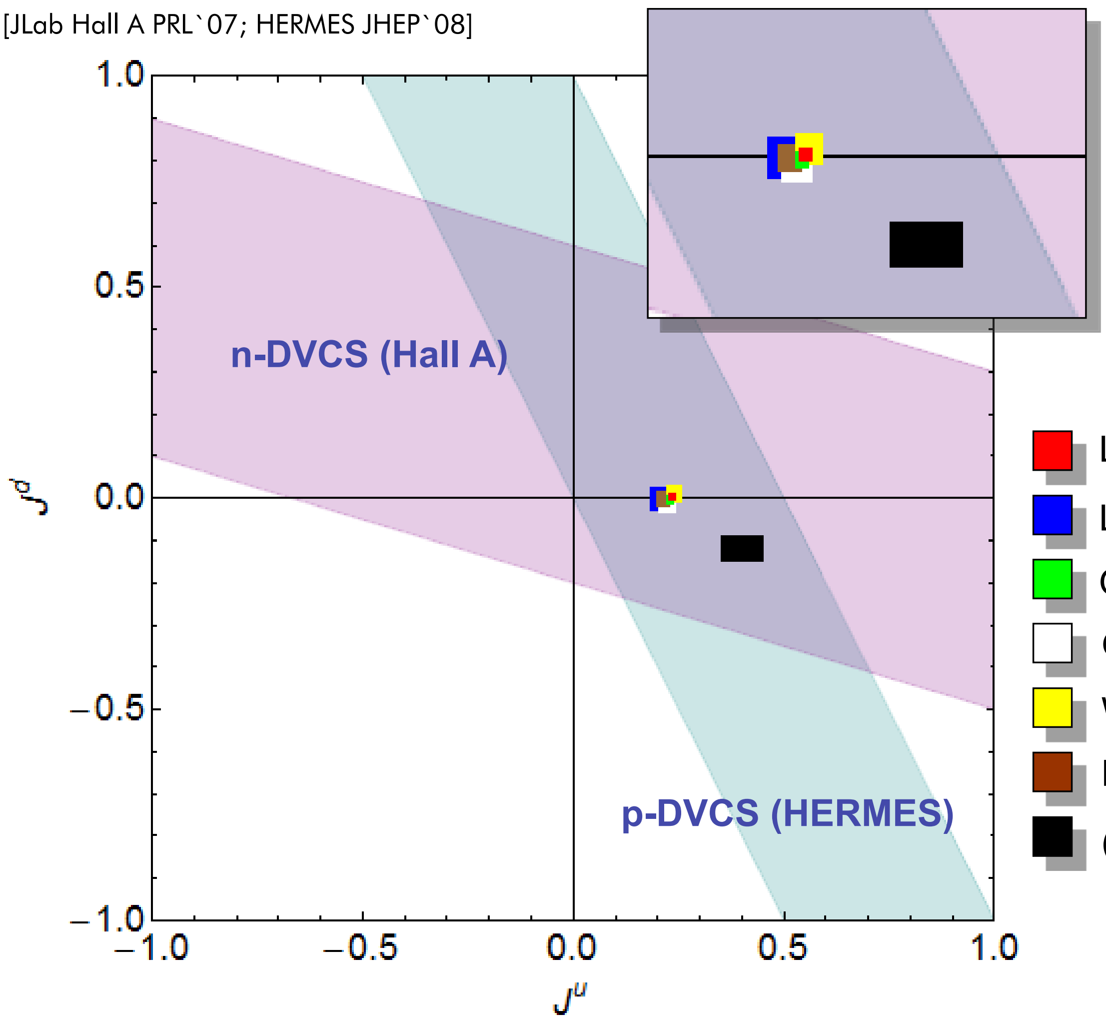
LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008); arXiv.1001.3620





Disconnected contributions neglected.

Origin of Nucleon Spin - II



Ph. Hagler, Menu 2010

- LHPC arXiv:1001.3620 (this work)
- LHPC PRD `08 0705.4295
- QCDSF (Ohtani et al.) 0710.1534
- Goloskokov&Kroll EPJC`09 0809.4126
- Wakamatsu 0908.0972
- DiFeJaKr EPJC `05 hep-ph/0408173
- (Myhrer&)Thomas PRL`08 0803.2775

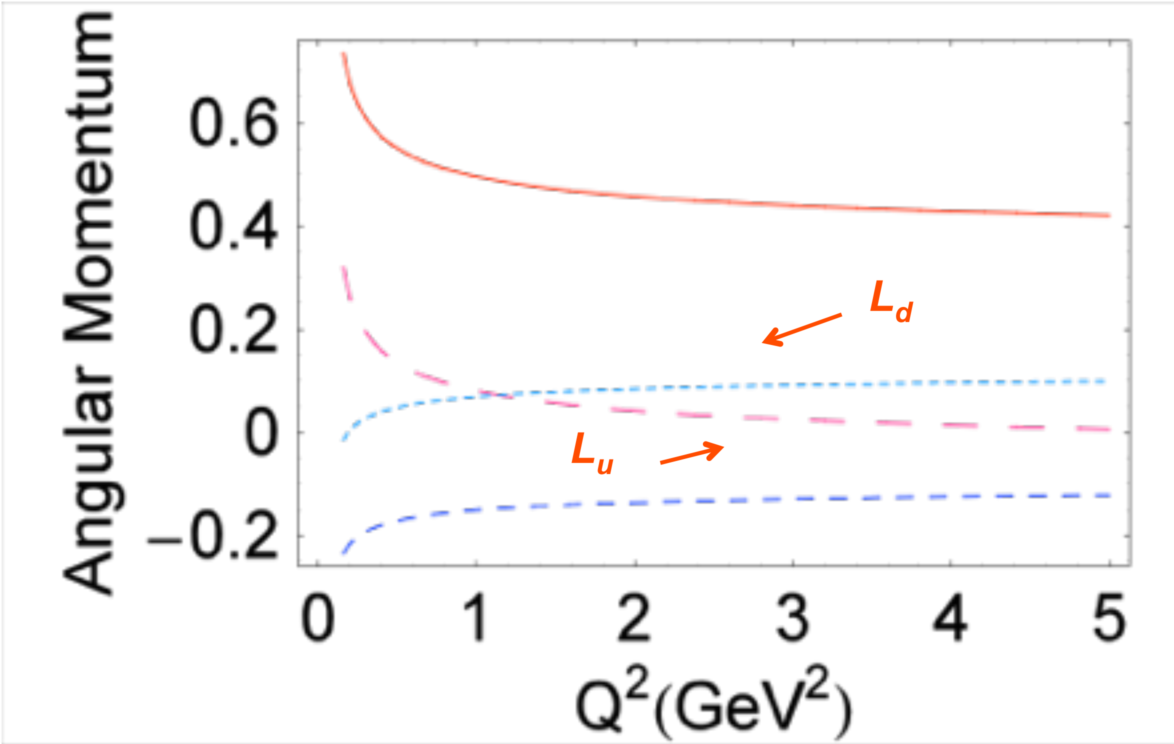
$\overline{MS}$ at 4 GeV^2

Origin of nucleon spin - III

"Missing spin": orbital angular momentum of quarks and anti-quarks

L_u +ve, L_d small, -ve at model scale

Myhrer & Thomas,
Phys.Lett.B663:302 (2008)



A.Thomas, Phys. Rev. Lett. 101:102003 (2008)

Gluon Contribution......

- **Can we go further?**

JI's sum rule

$$\frac{1}{2} = J^Q + J^G$$

We also have:

Jaffe, Manohar, NPB337, 509

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$$

← **Light-cone decomposition**

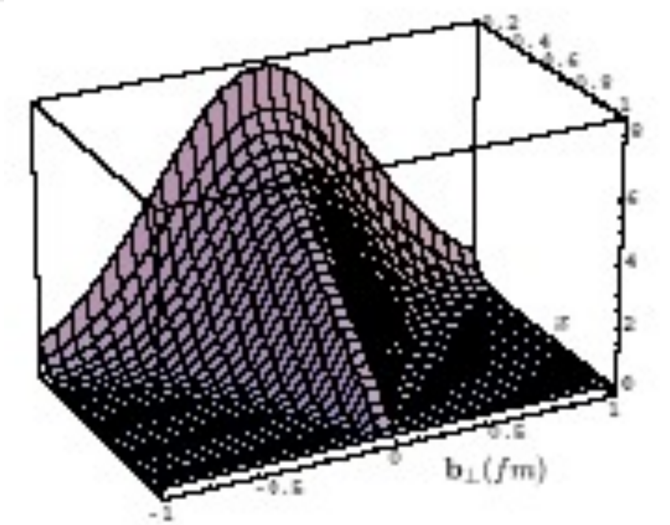
Cannot further decompose

$$L^g = J^G - \Delta G$$

Jaffe, hep-ph/0008038
Burkardt, Miller, Nowak, arXiv: 0812.2208

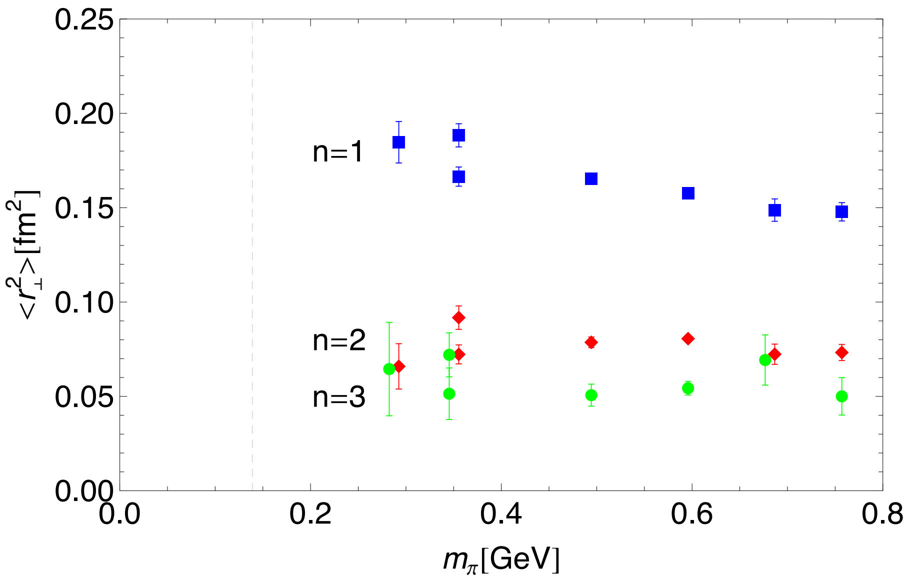
Transverse Structure

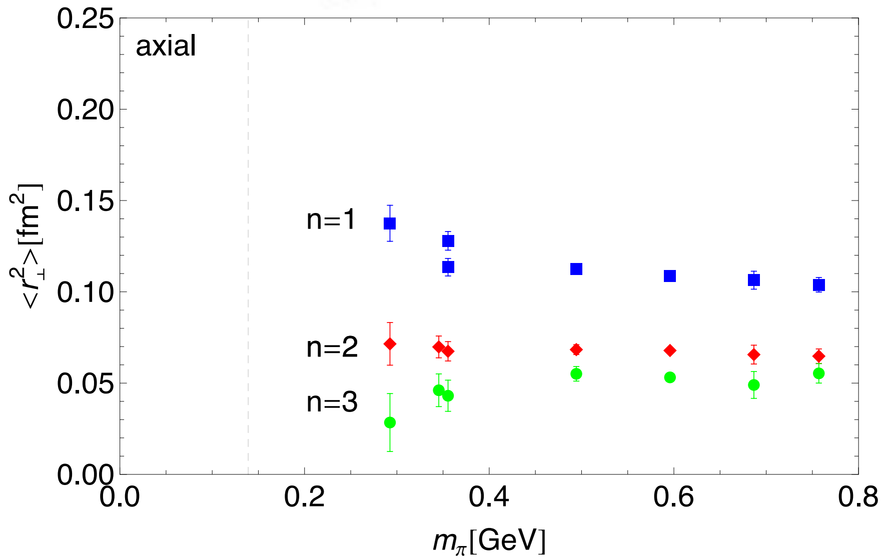
Lattice results consistent with narrowing of transverse size with increasing x ***Burkardt***



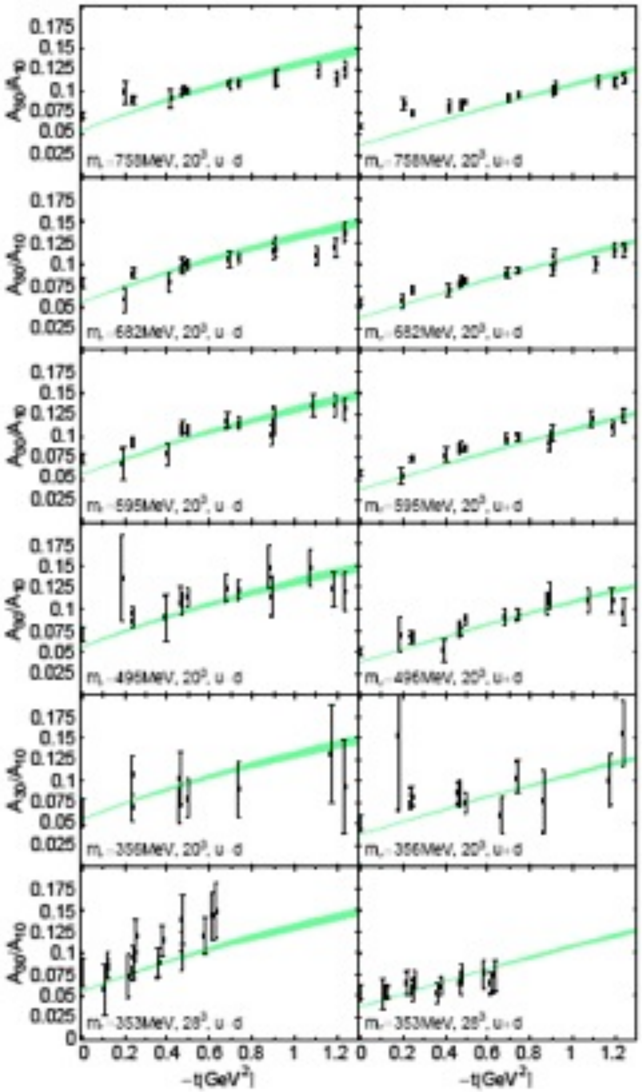
Flattening of GFFs with increasing n

$$A^q_{n0}(-\vec{\Delta}^2_\perp) = \int d^2 b_\perp \, e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \int_{-1}^{1} dx \, x^{n-1} q(x, \vec{b}_\perp)$$





Parametrizations of GPDs



Provide phenomenological guidance for GPD's

- *CTEQ, Nucleon Form Factors, Regge*

Comparison with *Diehl et al, hep-ph/0408173*

Important Role for LQCD

Transverse Spin in Nucleon

Measuring generalized form factors corresponding to tensor current gives provides information on transverse spin of nucleon

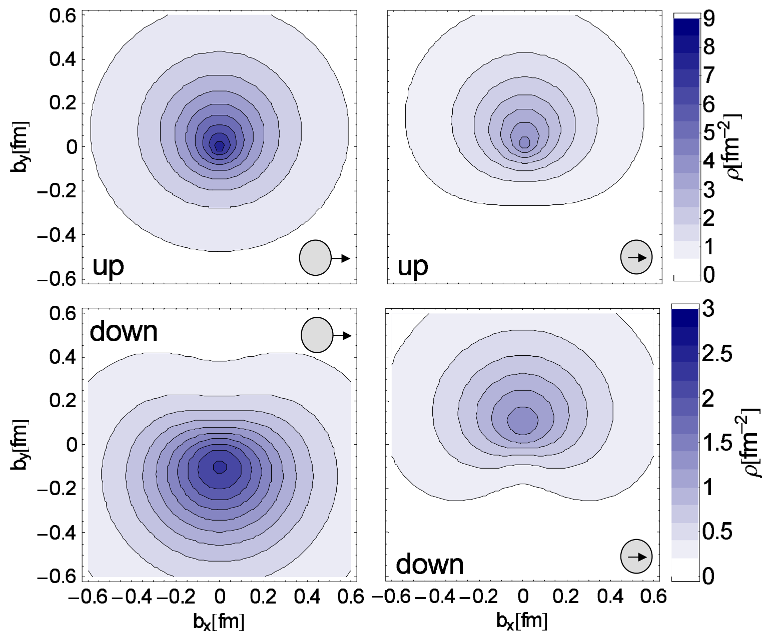
$$\langle P'\Lambda'|\mathcal{O}_T^{\mu\nu}|P\Lambda\rangle = \overline{u}(P',\Lambda')\Big\{\sigma^{\mu\nu}\gamma_5\Big(A_{T10}(t) - \frac{t}{2m^2}\widetilde{A}_{T10}(t)\Big) + \frac{\epsilon^{\mu\nu\alpha\beta}\Delta_\alpha\gamma_\beta}{2m}\overline{B}_{T10}(t) - \frac{\Delta^{[\mu}\sigma^{\nu]\alpha}\gamma_5\Delta_\alpha}{2m^2}\widetilde{A}_{T10}(t)\Big\}u(P,\Lambda)\ ,$$

QCDSF/UKQCD, PRL, 0612021

$$\mathcal{O}_T^{\mu\nu} = \bar{q}\sigma_{\mu\nu}\gamma_5 q$$

Lowest moment $B_{T10}(t)$

TMDs - Berni Musch, this session

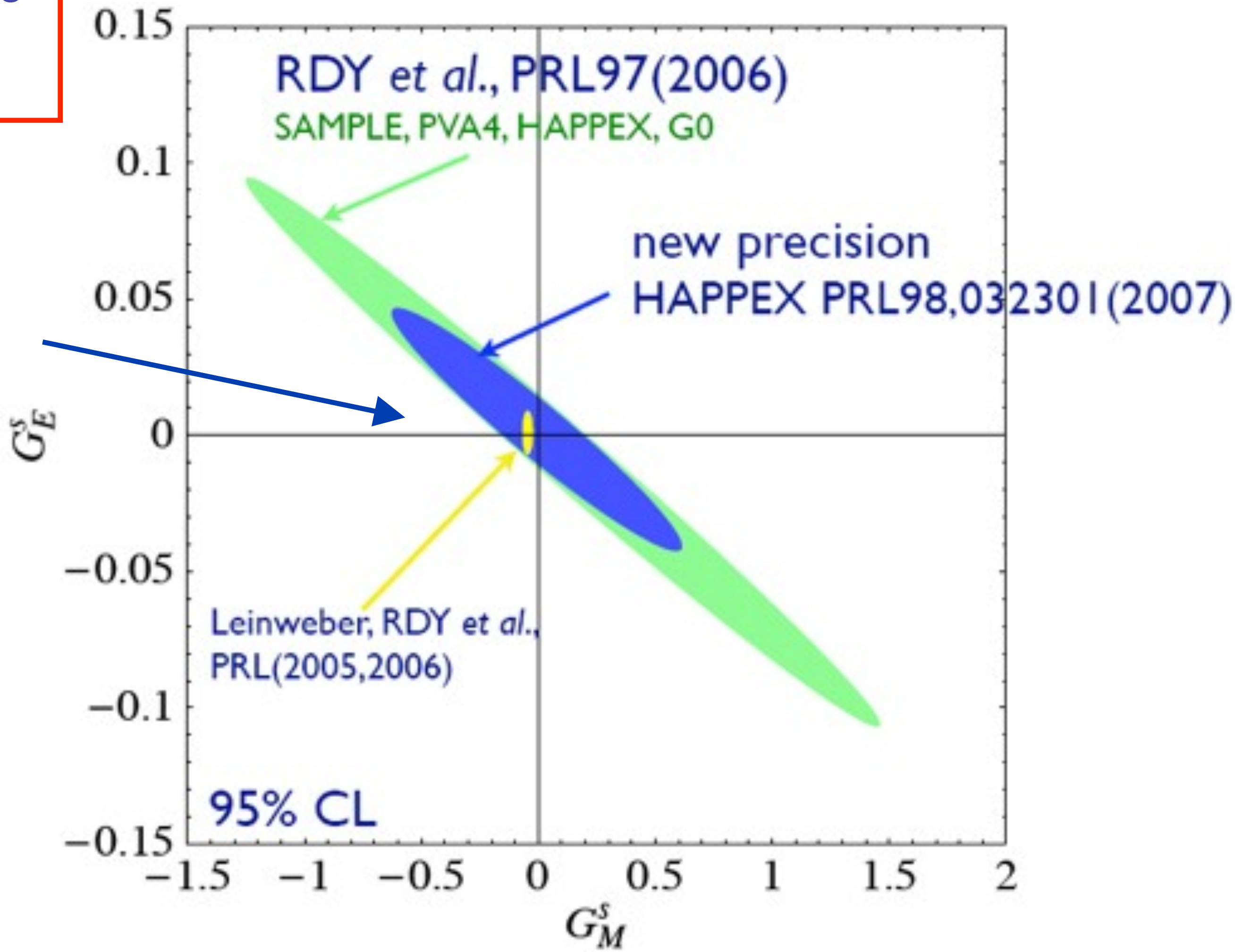


Flavor-Singlet Hadron Structure

Flavor-singlet: Disconnected Contributions

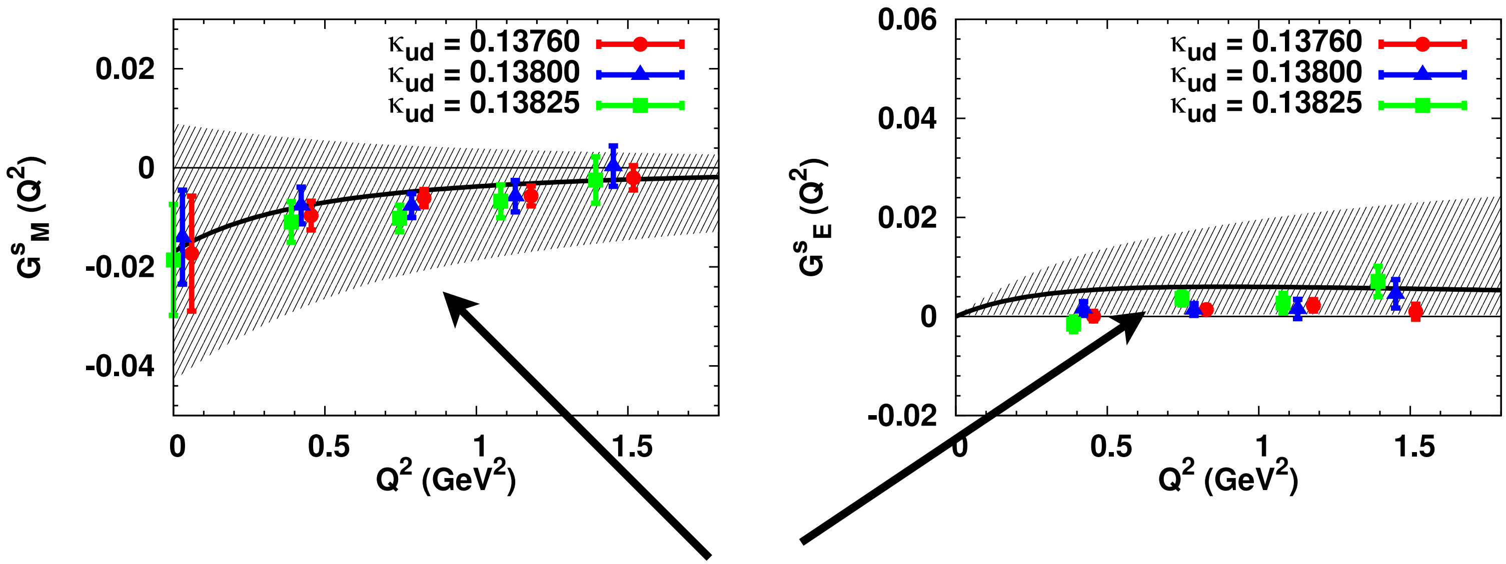
Strangeness contribution to electric and magnetic form factors.

Amalgam of Lattice QCD and Phenomenology by Leinweber *et al.*



Ab initio calculation

Doi et al. (ChQCD Collaboration),
arXiv:0910.2687, PRD79:094502,2009

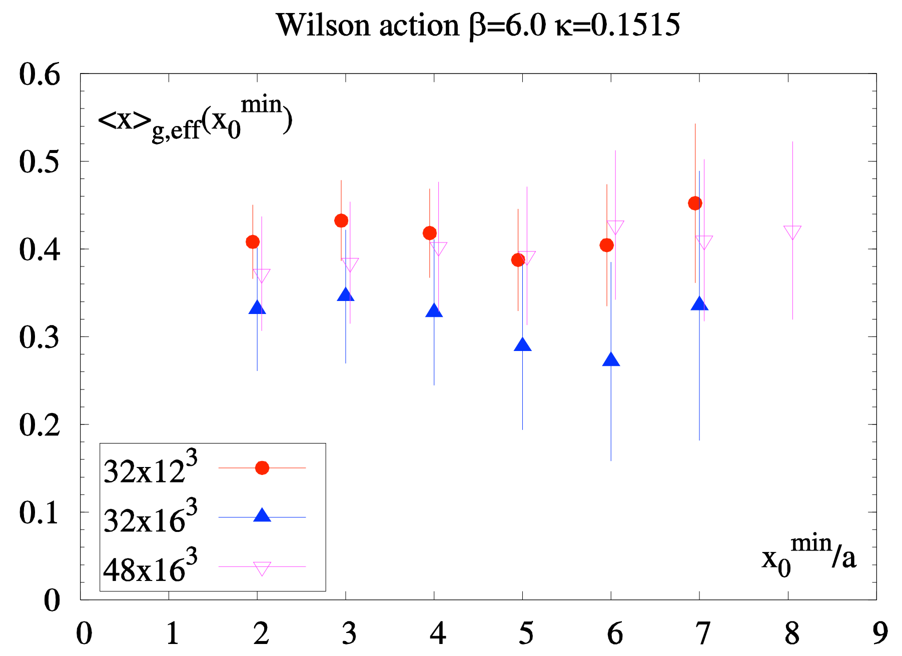


Uncertainties: statistical, Q^2 dependence, chiral extrapolation

$$G^s_M(0) = -0.017(25)(07)$$

Gluon Momentum Fraction in Pion

- **Flavour-singlet: mixing of quark and gluon contributions**
- **Notoriously difficult, but essential**
- **Improved operator $E^2 - B^2$: 40x increase in signal**
- **Normalize operator by ratio of entropy at finite T**



H. Meyer, J. Negele, PRD (2008)

$$\langle x \rangle_{\text{glue}}(\mu = 2\ \text{GeV}) = 0.37 \pm 8 \pm 12$$

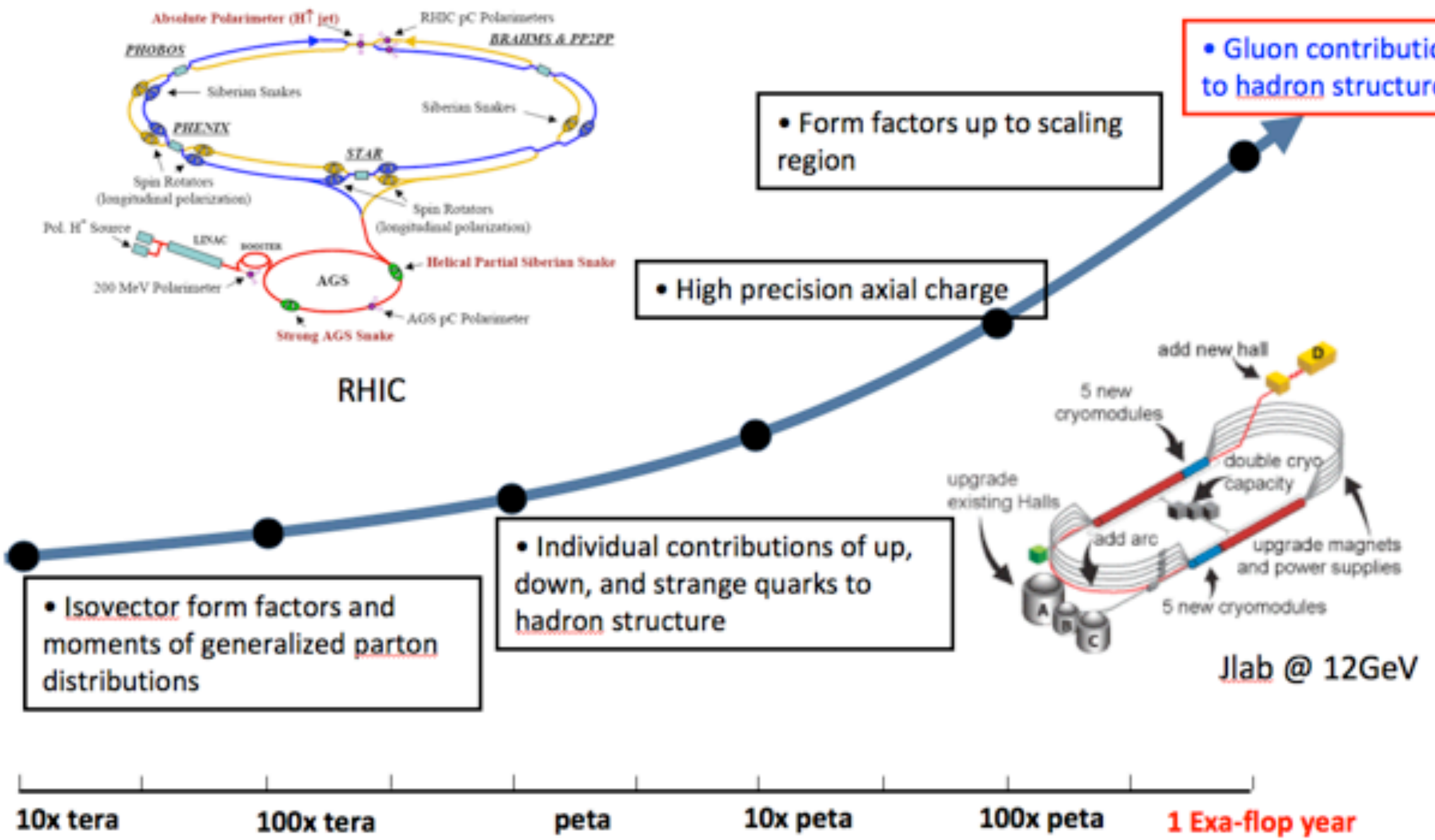
Momentum sum rule: $\langle x \rangle_{\text{glue}} + \langle x \rangle_{\text{quarks}} = 0.99 \pm 8 \pm 12$

Summary

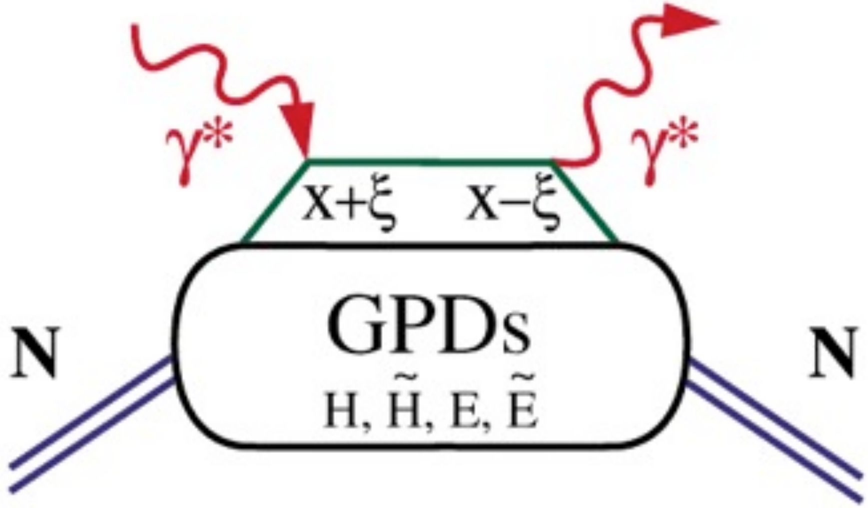
- **GPDs and TMDs are Drawing a three-dimensional picture of the Proton**
- **Increasing range of observables accessible to lattice calculations**
  - **GPDs:** *Orbital angular momentum*
  - **Flavor-singlet and *gluonic observables*.**
- **Lattice QCD calculations and experiment together can have greater predictive power than either alone; phenomenology of LQCD + Expt.**
- **Next talk..: transversity opening new avenues for lattice QCD.**

Lattice QCD Roadmap

Workshop on Extreme Computing, Jan. 2009

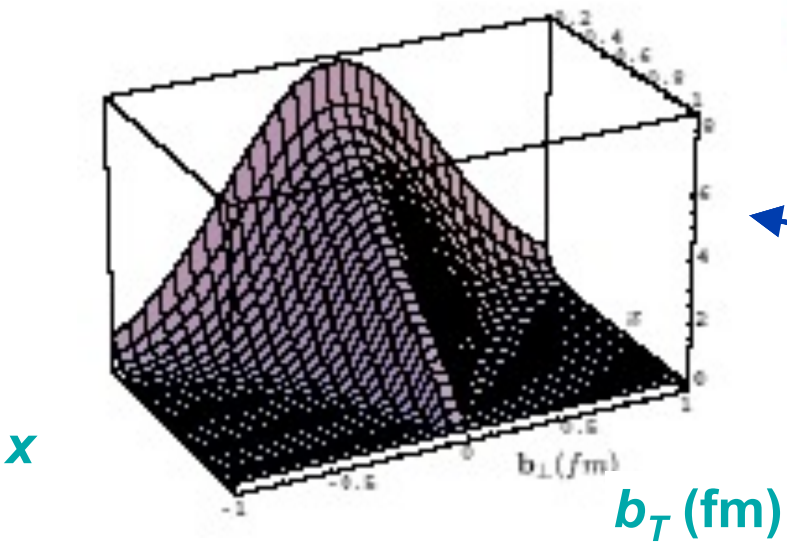


Transverse Distribution - I



• Lattice QCD can compute moments of GPDs and PDFs, and the *t*-dependence

$$A^q_{n0}(-\vec{\Delta}_\perp^2) = \int d^2 b_\perp \, e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \int_{-1}^{1} dx \, x^{n-1} q(x, \vec{b}_\perp)$$

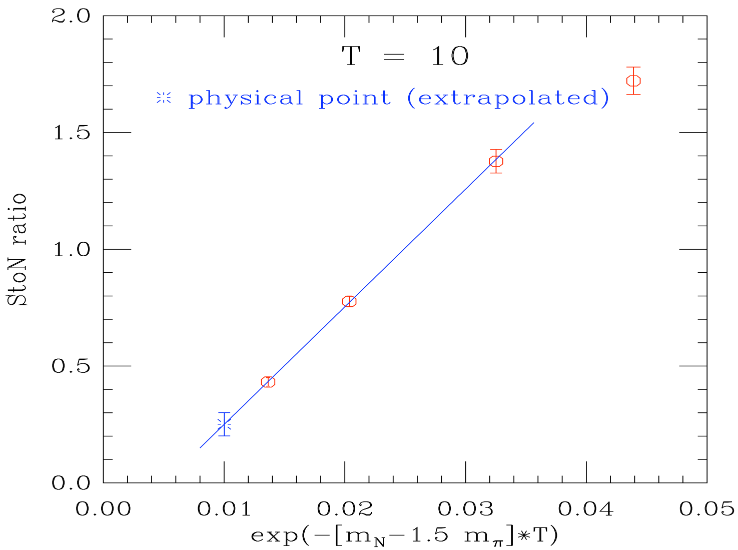


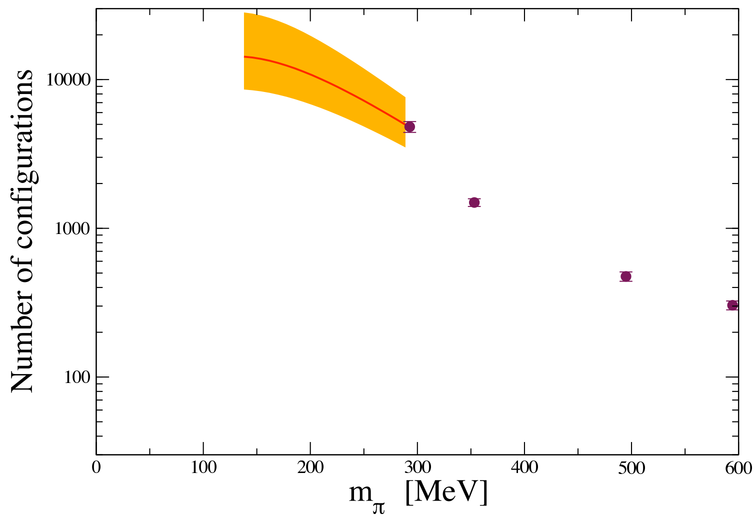
Compare to phenomenological models

Decrease slope : decreasing transverse size as x ! 1

Burkardt

Statistics for Hadron Structure





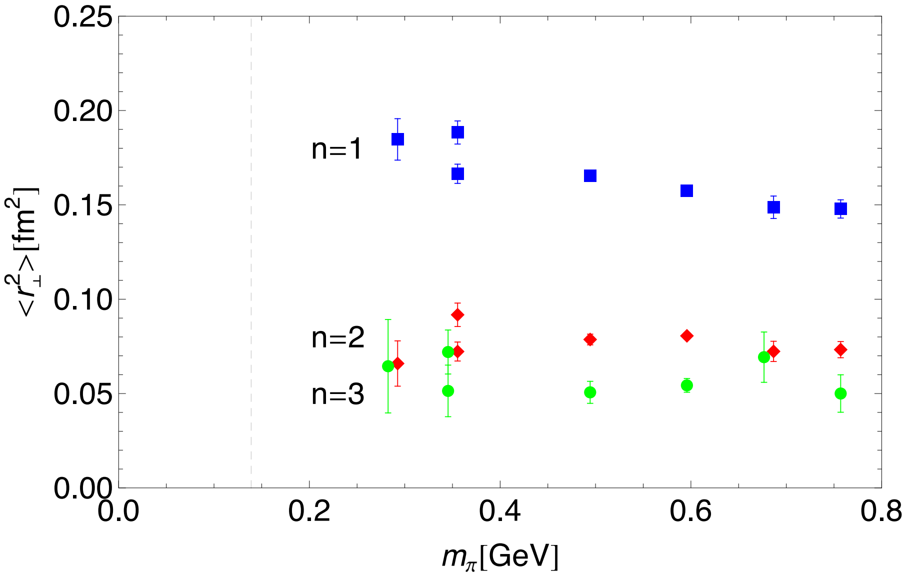
Increasing statistics in approach to physical quark mass: *more severe for baryons than mesons*

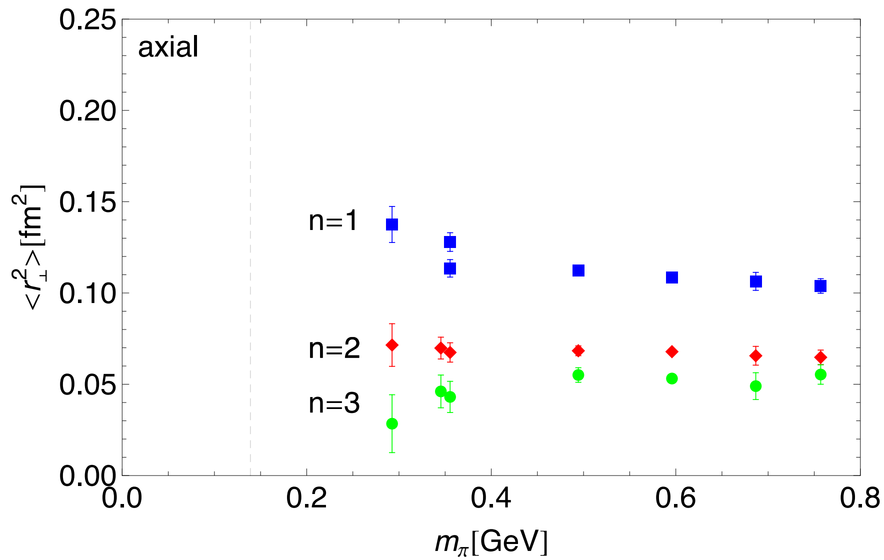
Transverse Distribution - II

Lattice results consistent with narrowing of transverse size with increasing x

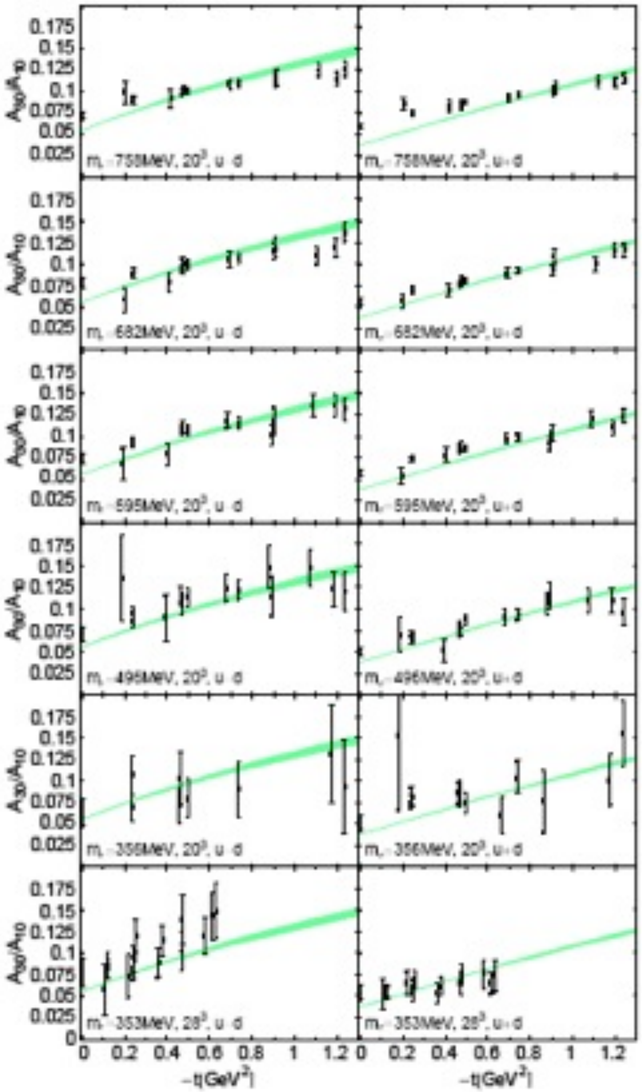
LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008)

Flattening of GFFs with increasing n





Parametrizations of GPDs



Provide phenomenological guidance for GPD's

- ***CTEQ, Nucleon Form Factors, Regge***

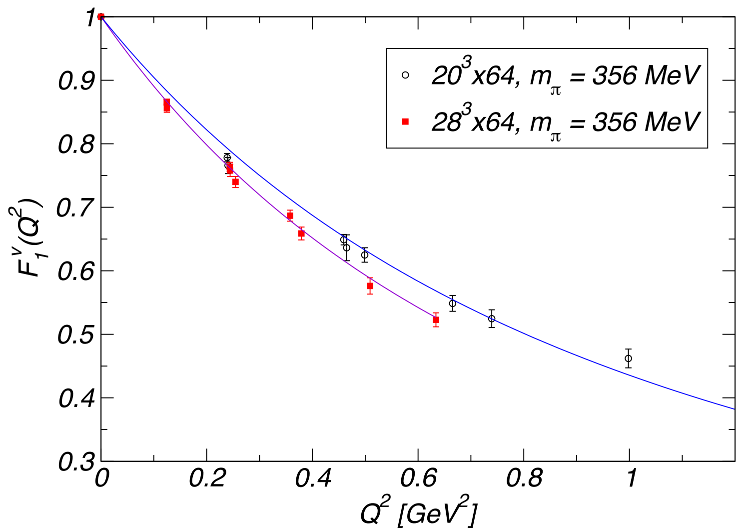
Comparison with *Diehl et al,* hep-ph/0408173

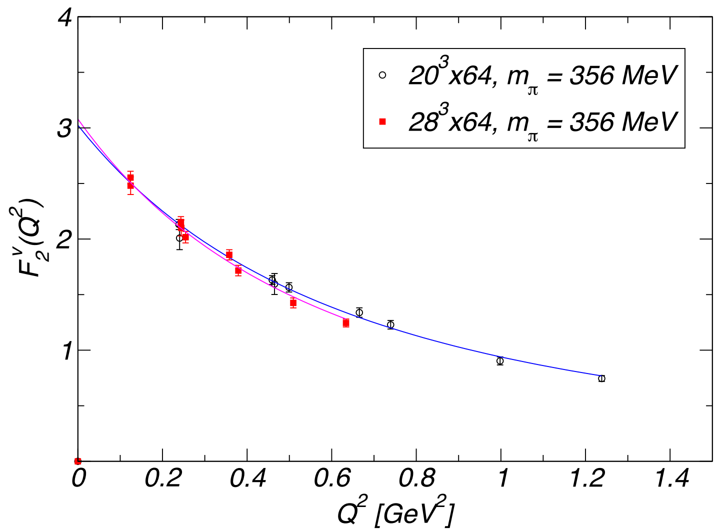
Important Role for LQCD

Nucleon Form Factors - II

LHPC, arXiv:1001:3620

T. Hemmert, HS2





Data well described by dipole form - but example of notable finite-volume effect